1 Judgment of conditional likelihood (or probability)

- Under many day-to-day circumstances you must estimate the probability of an event given that certain actions are taken.

2 Circumstances

- What are the chances of something serious if you don't go to the doctor for that cough?
- What are the chances of lead poisoning if you remove the paint vs. painting over it?
- What are the chances of getting into a clinical program if I get a C in this class?
- How likely am I to get AIDS if I have unprotected sex with this person?

3 Definition:
Conditional Probability

- The probability of an event given that another one has occurred.
- P(A|B) – probability of A given B.
- Venn Diagram

4

5 Dealing with negation

- P(A|not B) – probability of A given that B has not occurred.

6

7 Confusion of the inverse

- People often think that P(A|B) = P(B|A)

8 Examples:
P(A=brown|B=clear) = .20

9 Example

- You learn that the probability of someone who commits a heinous crime having a history of violent behavior is 70%.
- You are then told that someone has a history of violent behavior.
- What is the probability that they will commit a violent crime?
  - P(violent past | crime) = .70
  - P(crime | violent past) = ???

10 Venn Diagrams
P(A=violent past|B=crime) = .70

11 The Ratio Rule

- This is called the "ratio rule"
  - The ratio of P(A|B) to P(B|A) is equal to the ratio of the base rates of A and B, respectively.
- Thus, P(A|B) = P(B|A) iff P(A) = P(B)

12 ❑ Apply the ratio rule to the earlier examples….

13 ❑ The ratio rule and drug testing

- ✓ If P(positive | drug user) = .99,
  Then what is P(drug user | positive)?
- ✓ Depends on P(positive) and P(drug user)...
- ✓ If P(positive) = P(drug user)
  Then P(positive | drug user) =
  P(drug user | positive)
- ✓ But, if P(positive) is higher than P(drug user)
  Then P(pos | drug user) > P(drug user | pos)
  - How much higher? Depends on P(pos)/P(drug user)

14 ❑ Relative base rates are important!!!

- ✓ The base rate of each event is critical to identifying P(A|B) if you already know P(B|A).

15 ❑ Example (Eddy, 1982)

- ✓ You are told that the base rate of breast cancer is 1%.
- ✓ A physician runs a mammogram on a patient. This mammogram is known to be positive 85% of the time when the patient has cancer.
- ✓ The test comes up positive for the patient.

16 ❑ What is the probability of the patient having cancer?

- ✓ Average answer: about 75%
- ✓ Real answer: about 7 to 8%
  - We'll see why later….
- ✓ When asked why they chose the higher value, physicians typically report that they thought that:
  - P(positive | cancer) = P(cancer | positive)
  - But, this is only the case when:
    P(positive) = P(cancer)

17 ❑ Obvious example

- ✓ The probability of your being a woman given that you're pregnant is 1.0.
  - P(woman | pregnant) = 1.0
- ✓ Does that mean that the probability of being pregnant given that you're a woman is 1.0????
  - P(pregnant | woman) <<< 1.0.
  - Remember the ratio rule:
    - If P(woman) >>> P(pregnant)
      then P(woman | pregnant) >>> P(pregnant | woman).

18 ❑ Bayesian inference

- ✓ The normative (i.e. optimal) way of deciding a conditional probability is through *Bayesian inference*.

19 Bayes theorem

- ✓ $P(H|E) = [P(E|H) \cdot P(H)] / P(E)$
  - Where:
    - $P(E) = P(E|H) \cdot P(H) + P(E|\neg H) \cdot P(\neg H)$
- ✓ P(H|E) is the posterior probability
- ✓ P(H) is the prior probability or base rate

20 Example

- ✓ A taxicab was involved in a hit and run accident one night.
- ✓ Two cab companies, Blue and Green, operate in the city.

21 Example, cont.

- ✓ 85% of the cabs in the city are Green, 15% Blue, and a witness identified cab as Blue.
- ✓ Court tested the witness' ability to identify cab colors under appropriate visibility conditions
  - He/she made the correct identification 80% of the time (and were wrong 20% of the time).
- ✓ What was the probability that the cab involved was Blue rather than Green?

22 Result

- ✓ Participants said 80% likely it was Blue.
  - Recall confusion of the inverse?
    - P(Blue|said was blue) = P(said was blue|Blue)
- ✓ Bayes says 41% is the right answer.
- ✓ What?

23 Frequency format

24 Medical example:

- ✓ AIDS is found in .6% of the American population.
- ✓ Assume that doctors have developed a test for AIDS that is 99% effective.
  - That is 99% of the time when patient is positive the test is positive and 99% of the time when patient is negative the test is negative.
- ✓ A particular person is giving blood and tests positive: what's the probability of AIDS?

25 What is the probability that she has AIDS?

- ✓ Bayes theorem: 37.4%!

26 Change the base rate….

- ✓ Assume person is a homosexual
  - AIDS is found in 4.4% of this population.
- ✓ Probability is now:
  - 82.0% !
- ✓ Exercise left to the listener….

27 Ignoring the base rate

- ✓Recall: Base rate is the prior probability when no evidence.
- ✓The taxicab and AIDS examples are evidence of *ignoring base rates*.
- ✓People have a strong tendency to ignore the base rates and rely solely on evidence.

28 Medical decision making

- ✓Doctors are frequently just as guilty of this type of reasoning as you and me.
- ✓Eddy (1982)
  - 100 physicians given a scenario in which the base rate was low (1% for breast cancer)
  - The diagnosticity of the test (mammogram) was relatively low:
    - P(positive | tumor) = .8
    - P(negative | no tumor) = .9

29 Results

- ✓95 of the physicians estimated probability of breast cancer at about 75% (normative is around 7.5%).
- ✓They ignored the base rate.

30 Source of biased attitudes

- ✓Some people have been saying religion is at the root of terrorism.
  - WTC, bombing of abortion clinics, Ireland.…
- ✓As proof, they provide evidence that P(religious conviction | terrorism) is large.
- ✓But, what we care about is
  P(terrorism | religious conviction).

31 Bias, continued.…

- ✓Bayes theorem (as found in the ratio rule) says that base rates of these two matter:
  - P(religious) >>> P(terrorism)
  - Thus P(religious | terrorism) will of course be much greater than P(terrorism | religious).
  - Therefore, evidence of high P(religious | terrorism) doesn't tell us much.
- ✓People are ignoring base rates, and thus confusing the inverse.

32 Exceptions to ignoring the base rate

- ✓If people *experience* the differences in base rates, then they are much more likely to use them.
- ✓Christiansen-Szalanski & Bushyhead (1981)
  - Physicians
- ✓Goodie & Fantino (1995, 1996)
  - Students

33 Summary

- ✓People confuse the inverse
  - Believe that: P(A|B) = P(B|A) unless this is obviously false (pregnancy example)
  - Proper judgment depends on base rates.
    - P(A) and P(B) (see ratio rule)
- ✓Bayes theorem is a normative way to obtain P(A|B) given P(B|A).

✓Easier to use frequency format than the theorem.

✓People who experience the base rate are more likely to use it.

34 ❑ Judgment of conjoint probability

✓Definition: P(A and B) = P(A) • P(B)

✓Example

- Two coin flips.
- What's the probability of two tails?
  - P(Tails and Tails) =
    P(Tails on first) • P(Tails on second) =
    0.5 • 0.5 = .25

35 ❑ Example 2

✓Two die rolls.

✓What's the probability of snake eyes (2)?

- P(1 and 1) =
  P(1 on first) • P(1 on second) =
  1/6 • 1/6 = 1/36 = 2.8%
- Pretty straightforward, but…

36 ❑ Cohen, Chesnick, & Haran (1971)

✓Subjects asked to estimate the probability of winning a lottery.

- To win, you must pick right combination of two numbers.
- First number could be 1 to 5.
- Second number could be 1 to 5.

✓Real probability: 1/5 • 1/5 = 1/25= 4%.

✓Estimated probability: 30%

✓People overestimate small probabilities… (prospect theory)

37 ❑ Independent vs. Dependent events

✓Prior examples assumed probabilistic independence.

✓Many events are not independent

- Example, rate probability that Maria is a Democrat and voted for Kerry.

✓For *independent* events, P(A & B) = P(A) • P(B), but not for *dependent* events

- But, even for dependent events...
  - P(A and B) <= P(A) and P(A and B) <= P(B)
- People don't conform….

38 ❑ Example of non-conformance: Conjunction error

✓People will rate

- P(Democrat & voted for Kerry) >
  P(Democrat) or P(Kerry)

✓This reasoning violates basic probability rules.

✓Why?

- A heuristic (plausibility) causes overestimates of a conjunction probability.

39 ❑ Conditional Likelihood: Summary

- ✓Normative: Bayesian inference
- ✓Tendency to ignore the base rate
- ✓People reason more normatively when problem posed as frequencies rather than probabilities.
- ✓People do not reason properly about conjoint probabilities.
- ✓Next: An alternative theory for likelihood judgment by combining evidence.